

## APPENDIX 1

Approximation to CCR of logistic growth function

The signed curvature  $K$  of a plane curve given as  $g = f(t)$  is:

$$(5) \quad K = \frac{g''}{(1+g'^2)^{3/2}}$$

We made the assumption that the squared slope of the plane curve  $g(t)$  ( $gOt$ - plane) is smaller than one; i.e.:

$$(6) \quad K \approx g'' \text{ when } g'^2 \ll 1 \text{ and } 0 < \mu < 4/g_{\max}$$

The extreme points of the curvature change rate can be determined as the points where the first derivative of the rate of change in curvature has zero values; i. e., to find them, we have to solve the equation:

$$(7) \quad K'' = 0, (K'' - \text{second derivative of } K).$$

The solutions of the equation (7) in the case of a logistic model (Equation 2) are equations (4 a, b, c).

## APPENDIX 2.

Very often the logistic function is used in another form:

$$(8) \quad g(t) = \frac{c}{1+e^{a+bt}} + d$$

where  $t$  is time in days,  $g(t)$  is the growth function value at time  $t$ , and  $a, b, c, d$  are fitting parameters. Applying the CCR method to the logistics function (Equation 8), we obtain the growth metrics as:

$$(9a) \quad t_0 = \frac{-a}{b} + \frac{1}{b} \ln(5 - 2\sqrt{6})$$

$$(9b) \quad t_{inf} = \frac{-a}{b}$$

$$(9c) \quad t_f = \frac{-a}{b} + \frac{1}{b} \ln(5 + 2\sqrt{6})$$

But in this case, it is difficult to give a biological interpretation of the model parameters of the logistic function (8);  $a$  is positive and  $b$  is negative during growth. That is why we prefer to work with the logistics function in the form (2).